

What is claimed is:

1           1.       A method for reconstructing an image of a scattering medium, comprising:  
2                   directing energy into the scattering medium at a source location on the  
3       scattering medium;  
4                   measuring the energy emerging from the scattering medium at a detector  
5       location on the scattering medium;  
6                   selecting an initial guess of internal properties of the scattering medium;  
7                   predicting the energy emerging from the scattering medium using an  
8       equation of radiative transfer, wherein the prediction is a function of the initial guess;  
9                   generating an objective function based on a comparison of the prediction  
10       with the measurement;  
11                  generating a gradient of the objective function by a method of adjoint  
12       differentiation;  
13                  modifying the initial guess of the properties of the scattering medium  
14       based on the gradient of the objective function; and  
15                  generating an image representation of the internal properties of the  
16       scattering medium.

1           2.       The method according to claim 1, further comprising repeating the  
2       predicting of the energy emerging from the scattering medium based on the modified  
3       initial guess, generating the objective function and modifying the initial guess, until at  
4       least one of a predetermined number of repetitions has occurred and the objective  
5       function reaches a predetermined threshold.

1           3.       The method according to claim 1, wherein the prediction depends on the  
2       boundary conditions.

1           4.       The method according to claim 3, wherein the boundary conditions  
2       account for a refractive mismatch at an interface between the medium and at least one of  
3       the detectors and source.

1           5.       The method according to claim 1, wherein the prediction comprises an  
2       iterative process producing intermediate results.

1           6.       The method according to claim 5, wherein the intermediate results of the  
2       prediction are stored.

1           7.       The method according to claim 6, wherein generating the gradient of the  
2       objective function by adjoint differences uses the intermediate results of the prediction.

1           8.       The method according to claim 7, wherein generating the gradient  
2       comprises stepping backward through the intermediate results of the prediction.

1           9.       The method according to claim 1, wherein the equation of radiative  
2       transfer is time independent.

1           10.      The method according to claim 9, wherein the time independent equation  
2       of radiative transfer is:

$$\omega \nabla \Psi(\mathbf{r}, \omega) + (\mu_a + \mu_s) \Psi(\mathbf{r}, \omega) = S(\mathbf{r}, \omega) + \mu_s \int_0^{2\pi} p(\omega, \omega') \Psi(\mathbf{r}, \omega') d\omega'$$

where  $\Psi(\mathbf{r}, \omega)$  is the radiance at the spatial position  $\mathbf{r}$  directed into a unit solid angle  $\omega$ ,  $S(\mathbf{r}, \omega)$  is the energy directed into the medium at spatial position  $\mathbf{r}$  into a unit solid angle  $\omega$ ,  $\mu_s$  is the scattering coefficient,  $\mu_a$  is the absorption coefficient and  $p(\omega, \omega')$  is the scattering phase function.

11. The method according to claim 10, wherein the scattering phase function is:

$$p(\cos \theta) = \frac{1 - g^2}{2(1 + g^2 - 2g \cos \theta)^{3/2}}$$

where  $\theta$  is the angle between the two unit solid angles  $\omega$  and  $\omega'$ , and  $g$  is the anisotropy factor.

12. The method according to claim 1, wherein the equation of radiative transfer is time dependent.

13. The method according to claim 12, wherein the time dependent equation of radiative transfer is:

$$\frac{1}{c} \frac{\partial \Psi(\mathbf{r}, \omega, t)}{\partial t} = S(\mathbf{r}, \omega, t) - \omega \cdot \nabla \Psi(\mathbf{r}, \omega, t) - (\mu_a + \mu_s) \Psi(\mathbf{r}, \omega, t) + \mu_s \int_0^{2\pi} p(\omega, \omega') \Psi(\mathbf{r}, \omega', t) d\omega'$$

where  $\Psi(\mathbf{r}, \omega, t)$  is the radiance at the spatial position  $\mathbf{r}$  directed into a unit solid angle  $\omega$ ,  $S(\mathbf{r}, \omega, t)$  is the energy directed into the medium at spatial position  $\mathbf{r}$  into a

6 unit solid angle  $\omega$ ,  $\mu_s$  is the scattering coefficient,  $\mu_a$  is the absorption coefficient and  
7  $p(\omega, \omega')$  is the scattering phase function.

1 14. The method according to claim 13, wherein the scattering phase function  
2 is:

3 
$$p(\cos\theta) = \frac{1 - g^2}{2(1 + g^2 - 2g\cos\theta)^{3/2}}$$

4 where  $\theta$  is the angle between the two unit solid angles  $\omega$  and  $\omega'$ , and  $g$  is  
5 the anisotropy factor.

1 15. The method according to claim 1, wherein the properties include at least  
2 one of a scattering coefficient, an absorption coefficient, an anisotropy factor, and a  
3 scattering phase function.

1 16. The method according to claim 1, wherein the objective function is a  
2 normalized comparison of the predicted energy and the measured energy

1 17. The method according to claim 1, wherein the objective function is based  
2 on the normalized sum of the differences between the predicted energy and the measured  
3 energy for each source detector pair, wherein a source detector pair is formed between  
4 each source location and each detector location.

1 18. The method according to claim 1, wherein the objective function is:

$$\varphi = \frac{1}{2} \sum_i^m (P_i - M_i)^2$$

where  $M_i$  represents the actual measurements and the  $P_i$  represents the predicted measurements for each source defector pair  $i$ ,  $m$  is the number of source detector pairs, where a source detector pairs is formed between each source location and each detector location.

19. The method according to claim 1, further comprising minimizing the objective function.

20. The method according to claim 19, wherein minimizing the objective function includes a one dimensional line search.

21. The method according to claim 20, wherein the one dimensional line search is performed along a direction of the gradient of the objective function.

22. The method according to claim 20, wherein the one dimensional line search is performed along a gradient-dependent direction.

23. The method according to claim 1, wherein the energy comprises near infra-red energy.

1           24.     The method according to claim 1, wherein the scattering medium contains  
2 regions wherein the scattering coefficients are not substantially greater than the  
3 absorption coefficients.

1           25.     The method according to claim 1, wherein the scattering medium contains  
2 a low scattering region embedded in a high scattering region.

1           26.     The method according to claim 1, wherein the predicted energy is  
2 determined using finite element methods.

1           27.     The method according to claim 1, wherein the predicted energy is  
2 determined using finite difference methods.

1           28.     A method for imaging the spatial optical properties of tissue, comprising:  
2                   (a)     directing energy into the scattering medium at a source location on  
3 the tissue;  
4                   (b)     measuring the energy emerging from the scattering medium at a  
5 detector location on the tissue;  
6                   (c)     selecting and initial guess of the spatial optical properties of the  
7 tissue;  
8                   (d)     predicting the energy emerging from the tissue using an equation  
9 of radiative transfer in an iterative process, wherein the prediction is a function of the

10 initial guess and a refraction index mismatch at a boundary of the tissue, and the iterative  
11 process generates a plurality of intermediate predictions;

12 (e) generating an objective function based on a normalized  
13 comparison of the prediction with the measured energy emerging from the scattering  
14 medium;

15 (f) generating a gradient of the objective function by adjoint  
16 differentiation;

17 (g) modifying the initial guess of the spatial properties of the tissue  
18 based on the gradient of the objective function;

19 (h) repeating steps (d) through (g) until at least one of a threshold of  
20 modifications to the initial guess is reached and the objective function reaches a  
21 threshold; and

22 (j) generating an image representation of the spatial optical properties  
23 of the tissue.

1 29. A system for reconstructing an image of a scattering medium, comprising:  
2 a source for directing energy into the scattering medium at source location on the  
3 scattering medium;  
4 a detector for measuring the energy emerging from the scattering medium at a  
5 detector location on the scattering medium;  
6 an initial guess of internal properties of the scattering medium;  
7 means for predicting the energy emerging from the scattering medium using an  
8 equation of radiative transfer, wherein the prediction is a function of the initial guess;

9 means for generating an objective function based on a comparison of the  
10 prediction with the measurement;  
11 means for generating a gradient of the objective function by a method of adjoint  
12 differentiation;  
13 means for modifying the initial guess of the properties of the scattering medium  
14 based on the gradient of the objective function; and  
15 means for generating an image representation of the internal properties of the  
16 scattering medium.

1 30. The system according to claim 1, further comprising means for repeating  
2 the predicting of the energy emerging from the scattering medium based on the modified  
3 initial guess, generating the objective function and modifying the initial guess, until at  
4 least one of a predetermined number of repetitions has occurred and the objective  
5 function reaches a predetermined threshold.

1 31. The system according to claim 1, wherein the prediction depends on the  
2 boundary conditions.

1 32. The system according to claim 31, wherein the boundary conditions  
2 account for a refractive mismatch at an interface between the medium and at least one of  
3 the detectors and source.



1           33.     The system according to claim 1, wherein the prediction comprises an  
2     iterative process producing intermediate results.

1           34.     The system according to claim 33, wherein the intermediate results of the  
2     prediction are stored.

1           35.     The system according to claim 34, wherein generating the gradient of the  
2     objective function by adjoint differences uses the intermediate results of the prediction.

1           36.     The system according to claim 35, wherein generating the gradient  
2     comprises stepping backward through the intermediate results of the prediction.

1           37.     The system according to claim 1, wherein the equation of radiative  
2     transfer is time independent.

1           38.     The system according to claim 37, wherein the time independent equation  
2     of radiative transfer is:

$$\omega \nabla \Psi(\mathbf{r}, \omega) + (\mu_a + \mu_s) \Psi(\mathbf{r}, \omega) = S(\mathbf{r}, \omega) + \mu_s \int_0^{2\pi} p(\omega, \omega') \Psi(\mathbf{r}, \omega') d\omega'$$

4           where  $\Psi(\mathbf{r}, \omega)$  is the radiance at the spatial position  $\mathbf{r}$  directed into a unit solid  
5     angle  $\omega$ ,  $S(\mathbf{r}, \omega)$  is the energy directed into the medium at spatial position  $\mathbf{r}$  into a unit  
6     solid angle  $\omega$ ,  $\mu_s$  is the scattering coefficient,  $\mu_a$  is the absorption coefficient and  $p(\omega, \omega')$   
7     is the scattering phase function.

1           39.     The system according to claim 38, wherein the scattering phase function  
2     is:

$$3 \qquad p(\cos\theta) = \frac{1 - g^2}{2(1 + g^2 - 2g\cos\theta)^{3/2}}$$

4           where  $\theta$  is the angle between the two unit solid angles  $\omega$  and  $\omega'$ , and  $g$  is the  
5     anisotropy factor.

1           40.     The system according to claim 1, wherein the equation of radiative  
2     transfer is time dependent.

1           41.     The system according to claim 40, wherein the time dependent equation of  
2     radiative transfer is:

$$3 \qquad \frac{1}{c} \frac{\partial \Psi(r, \omega, t)}{\partial t} = S(r, \omega, t) - \omega \cdot \nabla \Psi(r, \omega, t) - (\mu_a + \mu_s) \Psi(r, \omega, t) + \mu_s \int_0^{2\pi} p(\omega, \omega') \Psi(r, \omega', t) d\omega'$$

4           where  $\Psi(\mathbf{r}, \omega, t)$  is the radiance at the spatial position  $\mathbf{r}$  directed into a unit solid  
5     angle  $\omega$ ,  $S(\mathbf{r}, \omega, t)$  is the energy directed into the medium at spatial position  $\mathbf{r}$  into a unit  
6     solid angle  $\omega$ ,  $\mu_s$  is the scattering coefficient,  $\mu_a$  is the absorption coefficient and  $p(\omega, \omega')$   
7     is the scattering phase function.

1           42.     The system according to claim 41, wherein the scattering phase function  
2     is:

$$3 \qquad p(\cos\theta) = \frac{1 - g^2}{2(1 + g^2 - 2g\cos\theta)^{3/2}}$$

4 where  $\theta$  is the angle between the two unit solid angles  $\omega$  and  $\omega'$ , and  $g$  is the  
5 anisotropy factor.

1 43. The system according to claim 1, wherein the properties include at least  
2 one of a scattering coefficient, an absorption coefficient, an anisotropy factor, and a  
3 scattering phase function.

1 44. The system according to claim 1, wherein the objective function is a  
2 normalized comparison of the predicted energy and the measured energy.

1 45. The system according to claim 1, wherein the objective function is based  
2 on the normalized sum of the differences between the predicted energy and the measured  
3 energy for each source detector pair, wherein a source detector pair is formed between  
4 each source location and each detector location.

1 46. The system according to claim 1, wherein the objective function is:

$$2 \quad \varphi = \frac{1}{2} \sum_i^m (P_i - M_i)^2$$

3 where  $M_i$  represents the actual measurements and  $P_i$  represents the predicted  
4 measurements for each source detector pair,  $m$  is the number of source detector pairs,  
5 where a source detector pairs is formed between each source location and each detector  
6 location.

1           47.     The system according to claim 1, further comprising minimizing the  
2     objective function.

1           48.     The system according to claim 47, wherein minimizing the objective  
2     function includes a one dimensional line search.

1           49.     The system according to claim 48, wherein the one dimensional line  
2     search is performed along a direction of the gradient of the objective function.

1           50.     The system according to claim 49, wherein the one dimensional line  
2     search is performed along a gradient-dependent direction.

1           51.     The system according to claim 50, wherein the energy comprises near  
2     infra-red energy.

1           52.     The system according to claim 1, wherein the scattering medium contains  
2     regions wherein the scattering coefficients are not substantially greater than the  
3     absorption coefficients.

1           53.     The system according to claim 1, wherein the scattering medium contains  
2     a low scattering region embedded in a high scattering region.

1           54.     The system according to claim 1, wherein the predicted energy is  
2     determined using finite element methods.

1           55.     The system according to claim 1, wherein the predicted energy is  
2     determined using finite difference methods.

1           56.     A system for imaging the spatial distribution of optical properties of  
2     tissue, comprising:

3           (a)     a source for directing energy into the scattering medium at a source  
4     location on the tissue;

5           (b)     a detector for measuring the energy emerging from the scattering medium  
6     at a detector location on the tissue;

7           (c)     an initial guess of spatial optical properties of the tissue;

8           (d)     means for predicting the energy emerging from the tissue using an  
9     equation of radiative transfer in an iterative process, wherein the prediction is a function  
10    of the initial guess and a refraction index mismatch at a boundary of the tissue, and the  
11    iterative process generates a plurality of intermediate predictions;

12          (e)     means for generating an objective function based on a normalized  
13    comparison of the prediction with the measured energy emerging from the scattering  
14    medium;

15          (f)     means for generating a gradient of the objective function by adjoint  
16    differentiation;

17 (g) means for modifying the initial guess of the spatial properties of the tissue  
18 based on the gradient of the objective function;

19 (h) means for repeating steps (d) through (g) until at least one of a threshold  
20 of modifications to the initial guess is reached and the objective function reaches a  
21 threshold; and

22 (j) means for generating an image representation of the spatial optical  
23 properties of the tissue.

1 57. Computer executable software code stored on a computer readable  
2 medium, the code for reconstructing an image of a scattering medium, comprising:  
3 code to direct energy into the scattering medium at a source location on  
4 the scattering medium;  
5 code to measure the energy emerging from the scattering medium at a  
6 detector location on the scattering medium;  
7 code to receive an initial guess of internal properties of the scattering  
8 medium;  
9 code to predict the energy emerging from the scattering medium using an  
10 equation of radiative transfer, wherein the prediction is a function of the initial guess;  
11 code to generate an objective function based on a comparison of the  
12 prediction with the measurement;  
13 code to generate a gradient of the objective function by a method of  
14 adjoint differentiation;

15 code to modify the initial guess of the properties of the scattering medium  
16 based on the gradient of the objective function; and  
17 code to generate an image representation of the internal properties of the  
18 scattering medium.

1 58. Computer executable software code stored on a computer readable  
2 medium, the code for imaging the spatial distribution of optical properties of tissue,  
3 comprising:

4 (a) code to direct energy into the scattering medium at a source location on the  
5 tissue;

6 (b) code to measure the energy emerging from the scattering medium at a detector  
7 location on the tissue;

8 (c) code to receive an initial guess of spatial optical properties of the tissue;

9 (d) code to predict the energy emerging from the tissue using an equation of  
10 radiative transfer in an iterative process, wherein the prediction is a function of the initial  
11 guess and a refraction index mismatch at a boundary of the tissue, and the iterative  
12 process generates a plurality of intermediate predictions;

13 (e) code to generate an objective function based on a normalized comparison of  
14 the prediction with the measured energy emerging from the scattering medium;

15 (f) code to generate a gradient of the objective function by adjoint differentiation;

16 (g) code to modify the initial guess of the spatial properties of the tissue based on  
17 the gradient of the objective function;

18 (h) code to repeat steps (d) through (g) until at least one of a threshold of  
19 modifications to the initial guess is reached and the objective function reaches a  
20 threshold; and  
21 (j) code to generate an image representation of the spatial optical properties of the  
22 tissue.

1 59. A computer readable medium having computer executable software code  
2 stored thereon, the code for reconstructing an image of a scattering medium, comprising:  
3 code to direct energy into the scattering medium at a source location on  
4 the scattering medium;  
5 code to measure the energy emerging from the scattering medium at a  
6 detector location on the scattering medium;  
7 code to receive an initial guess of internal properties of the scattering  
8 medium;  
9 code to predict the energy emerging from the scattering medium using an  
10 equation of radiative transfer, wherein the prediction is a function of the initial guess;  
11 code to generate an objective function based on a comparison of the  
12 prediction with the measurement;  
13 code to generate a gradient of the objective function by a method of  
14 adjoint differentiation;  
15 code to modify the initial guess of the properties of the scattering medium  
16 based on the gradient of the objective function; and



17 code to generate an image representation of the internal properties of the  
18 scattering medium.

1 60. A computer readable medium having computer executable software code  
2 stored thereon, the code for imaging the spatial distribution of optical properties of tissue,  
3 comprising:

4 (a) code to direct energy into the scattering medium at a source location on the  
5 tissue;

6 (b) code to measure the energy emerging from the scattering medium at a detector  
7 location on the tissue;

8 (c) code to receive an initial guess of spatial optical properties of the tissue;

9 (d) code to predict the energy emerging from the tissue using an equation of  
10 radiative transfer in an iterative process, wherein the prediction is a function of the initial  
11 guess and a refraction index mismatch at a boundary of the tissue, and the iterative  
12 process generates a plurality of intermediate predictions;

13 (e) code to generate an objective function based on a normalized comparison of  
14 the prediction with the measured energy emerging from the scattering medium;

15 (f) code to generate a gradient of the objective function by adjoint differentiation;

16 (g) code to modify the initial guess of the spatial properties of the tissue based on  
17 the gradient of the objective function;

18 (h) code to repeat steps (d) through (g) until at least one of a threshold of  
19 modifications to the initial guess is reached and the objective function reaches a  
20 threshold; and

